Loop Transformation Frameworks for Sparse Codes and Program Synthesis Opportunities

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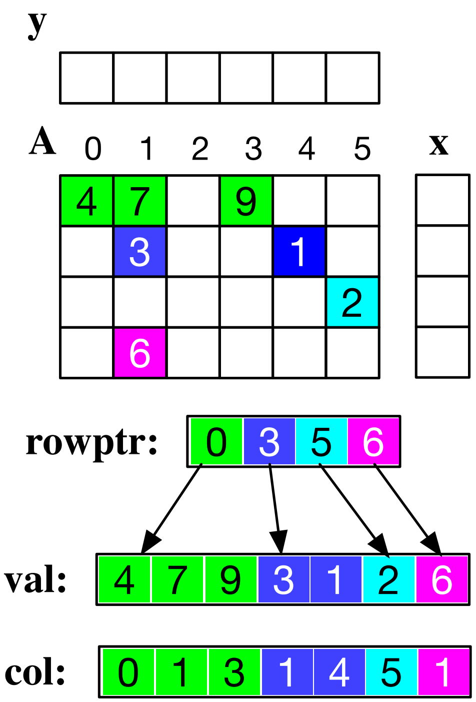
Sparse Codes are Hard to Optimize and Transform



y = A*x

```
// Dense matrix vector mult.
for (i = 0; i < N; i++) {
  for (j = 0; j < N; j++)
    y[i] +=  A[i][j] * x[j];
  }
}
```

```
// sparse matrix vector mult. (SpMV)
for (i=0; i<n; i++) {
  for(k=rowptr[i];k<rowptr[i+1];k++){
    y[i] += val[k]*x[col[k]];
  }
}
```

- Indirect accesses are slow
- Many different sparse formats
- Which sparse format is ideal depends on: algorithm, sparse structure, AND computation

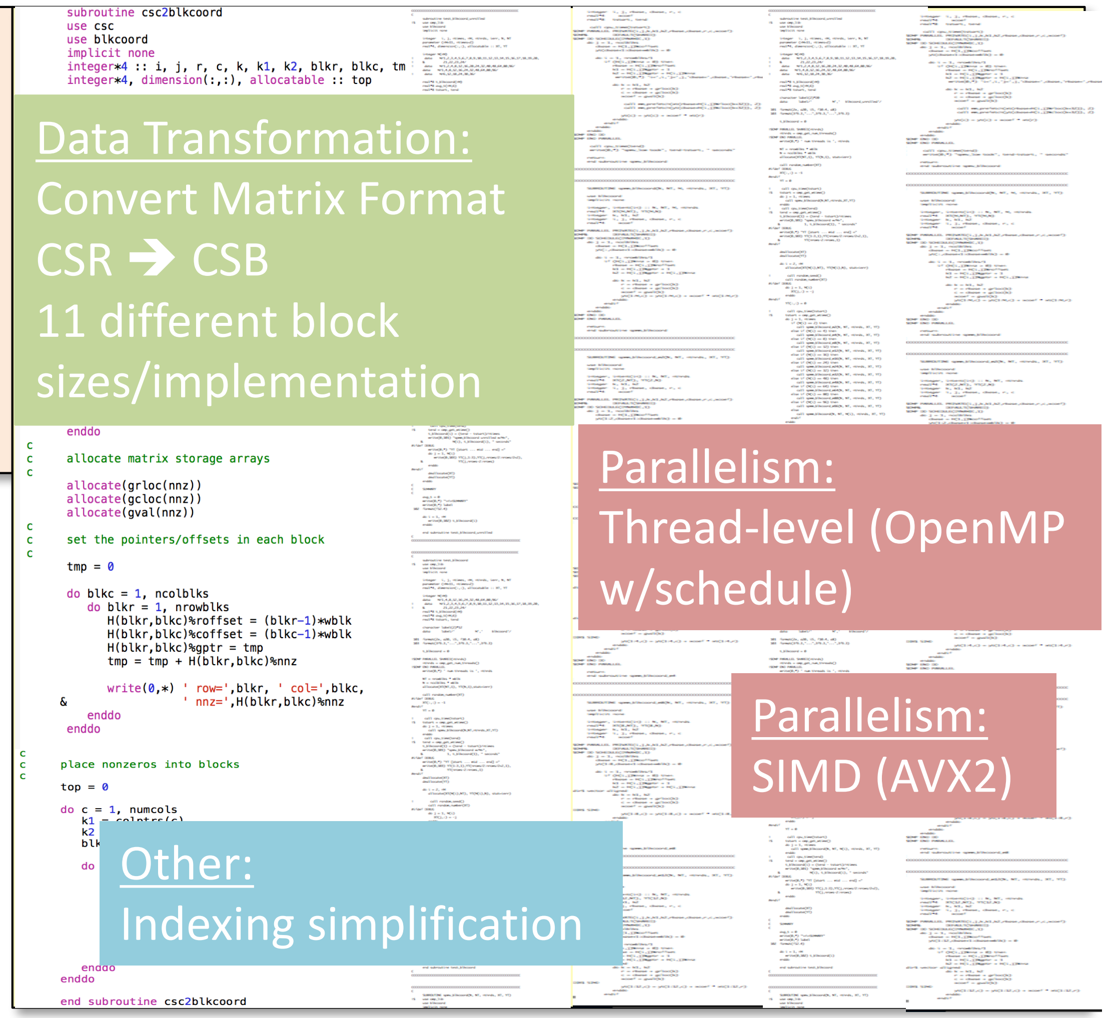
Current Approaches

- Developing new sparse formats and optimizations: HiCOO, sparse tiling, wavefront parallelization, ...
- Code generation from a DSL
  - Bernoulli compiler work
  - TACO work generates efficient implementations given a sparse tensor formats and a tensor expression
- Transforming existing code
  - Sparse Polyhedral Framework
  - CHiLL-I/E, scripting compiler for specifying inspector-executor transformations

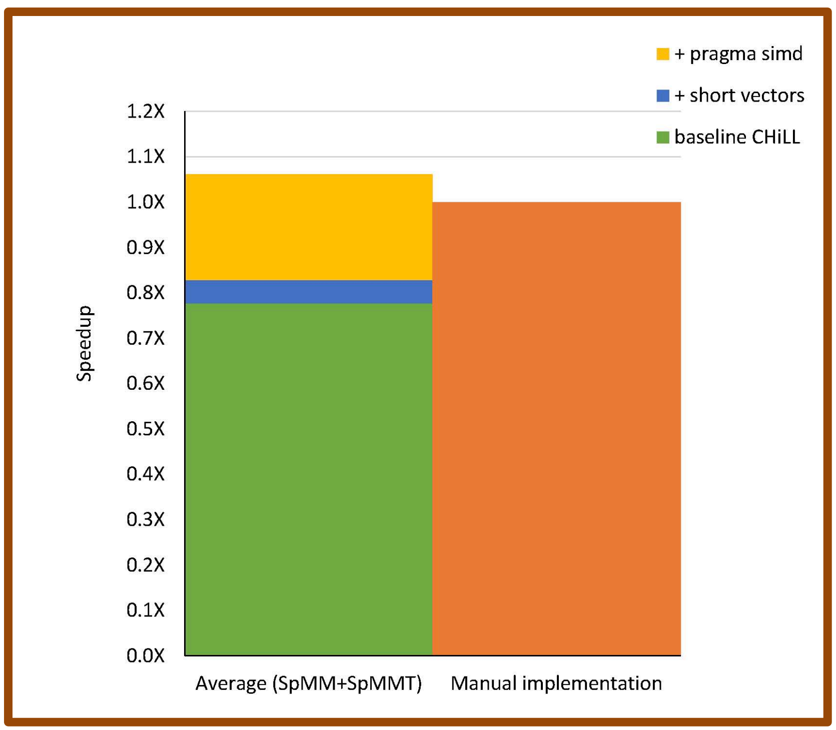
Transformation Example: SpMM from LOBPCG (NUCLEI)

```
/* SpMM from LOBCG on symmetric matrix */
for( i =0; i < n ; i ++) {
  for ( j = index [ i ]; j < index [ i +1]; j ++)
   for( k =0; k < m ; k ++);
      y [ i ][ k ]+= A [ j ]* x [ col [ j ]][ k ];
   /* transposed computation exploiting symmetry*/
   for ( j = index [ i ]; j < index [ i +1]; j ++)
      for( k =0; k < m ; k ++)
         y [ col [ j ]][ k ]+= A [ j ]* x [ i ][ k ];
}
```

Code A: Multiple SpMV computations (SpMM), 7 lines of code

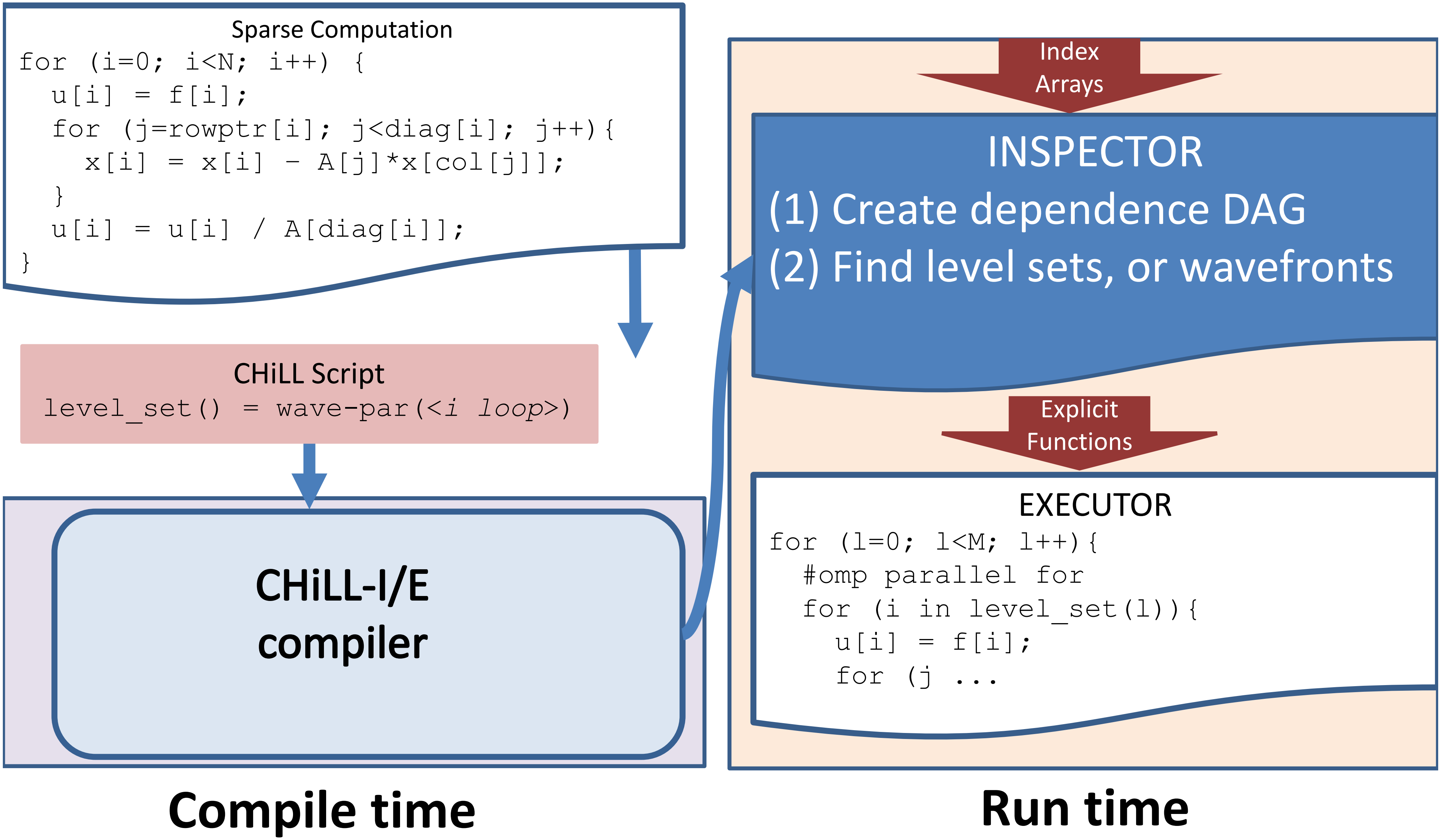


Code B: Manually-optimized SpMM from LOBCG, 2109 lines of code



Take-away message: Compiler-optimized Code A faster than manual Code B!

CHiLL-I/E: Inspector-Executor Transformations



Opportunities to Leverage Synthesis Tools?

- Constraint-solving-based synthesis techniques
  - Polyhedral model uses Farkas lemma to derive scheduling constraints from data dependences
  - Sparse Polyhedral Framework can produce constraints for the uninterpreted functions the inspector must produce at runtime

- Run-time realization of uninterpreted functions
  - Could be synthesized to specialize for usage
  - Data structure synthesis tools like Cozy

Deriving constraints for uninterpreted functions

- Constraint-based data dependence analysis
- Transformations introduce new uninterpreted functions and modify data dependences
- Convert data dependence relations into constraints on uninterpreted functions

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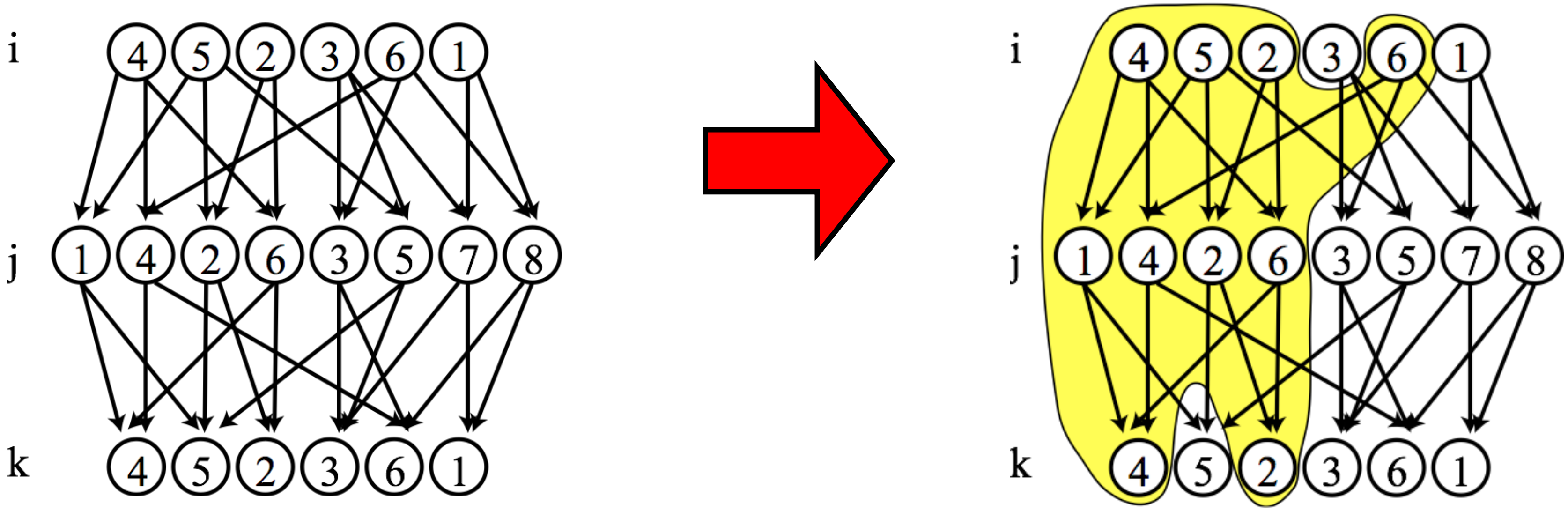
Constraint-Based Data Dependence Analysis of Sparse Computation

```
for (int j=0; j<n; j++){
  x[j] = x[j] / Lx[colPtr[j]];
  for(int p=colPtr[j]+1; p<colPtr[j+1]; p++){
    x[row[p]] = x[[row[p]] - Lx[p] * x[j];
```

$$\{[j,p] \rightarrow [j',p'] : \overbrace{j = j' \wedge p < p'}^{lexicographical\ Ordering} \wedge \overbrace{row(p) = j'}^{Array\ Access\ Equality} \wedge$$

$$\underbrace{0 \leq j, j' < n \wedge colPtr(j) < p < colPtr(j+1) \wedge colPtr(j') < p' < colPtr(j'+1)\}}_{Loop\ Bounds}$$

Example Transformation Introducing an Uninterpreted Function



$$T_{F_1 \to F_2} = \{[s,0,i] \to [s,0,t,0,i] \mid t = \Theta(0,i)\}$$
$$\cup \ \{[s,1,j] \to [s,0,t,1,j] \mid t = \Theta(1,j)\} \cdots$$

$$F_1 = \{[s,0,i]\} \cup \{[s,1,j]\} \cup \{[s,2,k]\}$$

$$F_2 = \{[s,0,t,0,i] \mid t = \Theta(0,i)\} \cup \{ [s,0,t,1,j] \mid t = \Theta(1,j)\} \cdots$$

Transformed Dependences Need to be Lexicographically Non-Negative

$$D_{I_0 \to J_0} = \{[s,0,i] \to [s,1,j] \quad | \quad i = l(j) \vee \; i = r(j)\}$$

$$\begin{aligned} T_{F_1 \to F_2} \quad = & \quad \{[s,0,i] \to [s,0,t,0,i] \mid t = \Theta(0,i)\} \\ \cup & \quad \{[s,1,i] \to [s,0,t,1,j] \mid t = \Theta(1,j)\} \cdots \end{aligned}$$

$$\begin{aligned} D_{I_0 \to J_0} = \{[s,0,t_1,0,i] \to [s,0,t_2,1,j] \quad | \quad & (t_1 = \Theta(0,i) \wedge t_2 = \Theta(1,j) \wedge i = l(j)) \\ & \vee (t_1 = \Theta(0,i) \wedge t_2 = \Theta(1,j) \wedge i = r(j))\} \end{aligned}$$

Constraints Derived from Dependence

$$D_{I_0 \to J_0} = \{[s,0,t_1,0,i] \to [s,0,t_2,1,j] \quad | \quad (t_1 = \Theta(0,i) \wedge t_2 = \Theta(1,j) \wedge i = l(j)) \\ \vee (t_1 = \Theta(0,i) \wedge t_2 = \Theta(1,j) \wedge i = r(j))\}$$

⇓

$$\forall s, t_1, t_2, i, j \ : \ (i = l(j) \vee i = r(j)) \Rightarrow \Theta(0,i) \leq \Theta(1,j)$$

If iteration i must be executed before iteration j, then iteration i must be in the same or earlier tile than j.

Summary: Synthesis and Transformed Sparse Codes

- Use dependence analysis of original code and inspector-executor transformations to create constraints
- Remains to be seen how these constraints can be used to synthesize inspector code
- Use data structure synthesis to generate specialized implementations of run-time realizations of uninterpreted functions (not discussed)